13 - ශ්‍රේණිය - 2023

සංයුක්ත ගණිතය - I

උත්තර මැද පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව.

① $f(n) = 3.5^{2n+1} + 2^{3n+1}$

$n=1$ අවස්ථාව

$f(1) = 3.5^3 + 2^4 = 391 = 17(23)$ 5

391 , 17 න් බෙදේ.

$n=1$ ට ප්‍රතිඵලය සත්‍ය වේ.

$n=p$ විට ප්‍රතිඵලය සත්‍ය යැයි උපකල්පනය කරමු. $(p \in \mathbb{Z}^+)$

$f(p) = 3.5^{2p+1} + 2^{3p+1}$ 5

$3.5^{2p+1} + 2^{3p+1} = 17k \quad (k \in \mathbb{Z}^+)$

$n=p+1$ අවස්ථාව

$f(p+1) = 3.5^{2p+3} + 2^{3p+4}$

$= 3.5^2.5^{2p+1} + 2^3.2^{3p+1}$

$= 75.5^{2p+1} + 8.2^{3p+1}$

$= 25(5^{2p+1}.3 + 2^{3p+1}) - 17.2^{3p+1}$ 5

$= 25.17k - 17.2^{3p+1}$

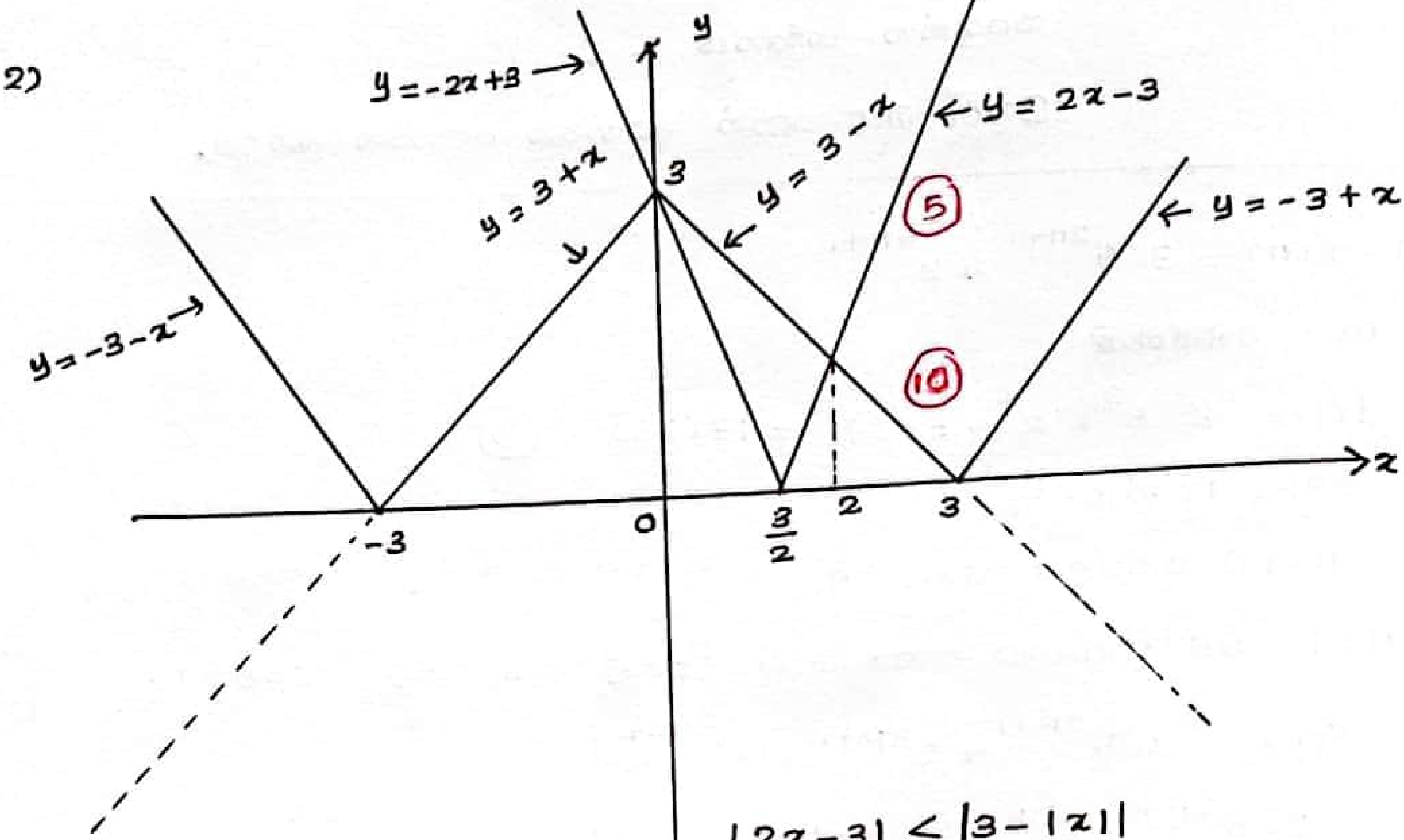
$= 17(25k - 2^{3p+1})$ 5

$n=p$ ට ප්‍රතිඵලය සත්‍ය යැයි උපකල්පනයෙන් $n=p+1$ ට ප්‍රතිඵලය සත්‍ය වේ. $n=1$ ට ප්‍රතිඵලය සත්‍ය බව පෙන්වා ලදී.

∴ සියලු $n \in \mathbb{Z}^+$ සඳහා ගණිත අභ්‍යුහන මූලධර්මයෙන් ප්‍රතිඵලය සත්‍ය වේ. 5

$\boxed{25}$

(2)



$|2x-3| \le |3-|x||$

ඡේදන ලක්ෂ්‍යය

$2x-3 = 3-x$

$3x = 6$

$x = 2$ (5)

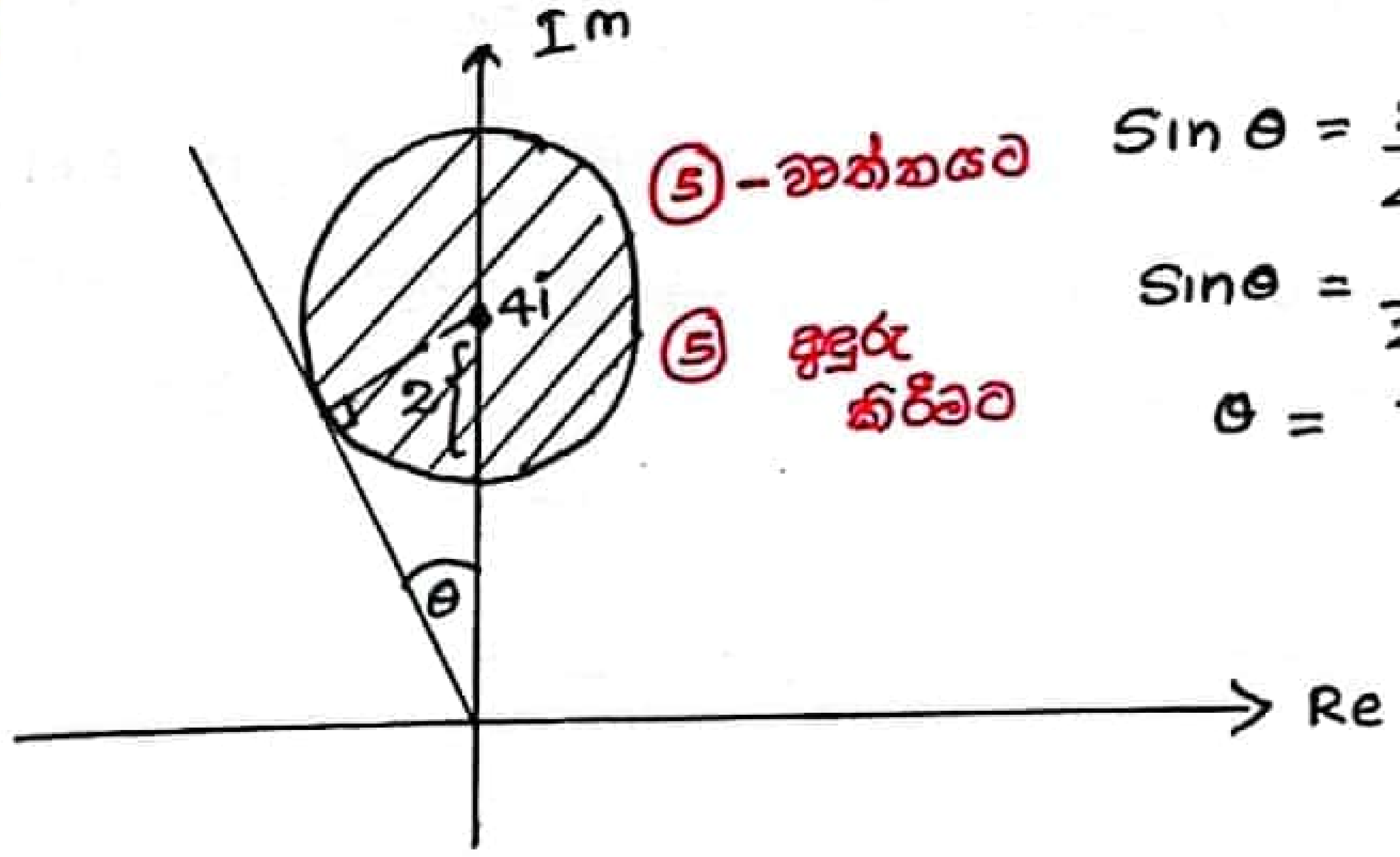
$0 \le x \le 2$ (5)

25

(3) $|\bar{z}+4i| \le 2$

$|z-4i| \le 2 \quad (|z| = |\bar{z}|)$

(5)



(5) - වෘත්තයට

(5) අඳුරු කිරීමට

$\sin\theta = \frac{2}{4}$

$\sin\theta = \frac{1}{2}$

$\theta = \pi/6$ (5)

$\text{Arg}(z) = \pi/2 + \pi/6$

$= 2\pi/3$ (5)

25

(4) $(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r$

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 1:7:42$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{1}{7} \quad 5 \qquad\qquad \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{7}{42} \quad 5$$

$$\frac{\dfrac{n!}{(n-r+1)!\,(r-1)!}}{\dfrac{n!}{(n-r)!\,r!}} = \frac{1}{7} \qquad\qquad \frac{\dfrac{n!}{(n-r)!\,r!}}{\dfrac{n!}{(n-r-1)!\,(r+1)!}} = \frac{1}{6}$$

$$\frac{(n-r)!\,(r-1)!\,r}{(n-r)!\,(n-r+1)(r+1)!} = \frac{1}{7} \qquad\qquad \frac{r!\,(r+1)\,(n-r-1)!}{(n-r-1)!\,(n-r)\,r!} = \frac{1}{6}$$

$$6r+6 = n-r \quad 5$$

$$7r = n-6 \rightarrow (2)$$

$$7r = n-r+1. \quad 5$$

$$8r = n+1 \longrightarrow (1)$$

$(1)-(2)$ $\qquad r = 7$ // $\qquad n = 55$ //

5 පිළිතුරු දෙකටම $\qquad \boxed{25}$

(5) $$\lim_{x\to 0} \frac{x\sin(x^2)}{\sin x\,(1-\sqrt{\cos x})}$$

$$\lim_{x\to 0} \frac{x\sin(x^2)}{\sin x\,(1-\sqrt{\cos x})} \times \frac{1+\sqrt{\cos x}}{1+\sqrt{\cos x}} \quad 5$$

$$\lim_{x\to 0} \frac{1}{\dfrac{\sin x}{x}} \cdot \lim_{x\to 0} \frac{\sin(x^2)}{1-\cos x} \times \lim_{x\to 0} 1+\sqrt{\cos x}$$

5

$$1 \times \lim_{x\to 0} \frac{\sin(x^2)}{2\sin^2\left(\frac{x}{2}\right)} \times 2$$

$$\frac{2}{2} \times \lim_{x\to 0} \frac{\sin x^2}{x^2} \times \lim_{x\to 0} \frac{1}{\dfrac{\sin^2 \frac{x}{2}}{x^2}} \qquad \boxed{25}$$

5

$$\lim_{x^2\to 0} \frac{\sin x^2}{x^2} \times \lim_{\frac{x}{2}\to 0} \times \frac{1}{\left(\dfrac{\sin x/2}{x/2}\right)^2} \times 4 = 1 \times \frac{1}{1} \times 4 = 4 //$$

5 $\qquad$ 5

(6) $V = \pi \int_0^1 \left[\dfrac{x^{3/2}}{(1+x^2)^2}\right]^2 dx$ 5

$\dfrac{dv}{dx} = 2x(1+x^2)^{-2}$

$v = \dfrac{-1}{(1+x^2)}$

$= \pi \int_0^1 \dfrac{x^3}{(1+x^2)^2}\, dx$

$u = x^2$

$\dfrac{du}{dx} = 2x.$

$= \dfrac{\pi}{2} \int_0^1 x^2 . 2x\, (1+x^2)^{-2} dx$

$= \dfrac{\pi}{2}\left[x^2 . \left(\dfrac{-1}{1+x^2}\right)\Big|_0^1 - \int_0^1 \dfrac{-1}{1+x^2} . 2x . dx \right]$ 5

$= \dfrac{\pi}{2}\left[-1 + \int_0^1 \dfrac{2x}{1+x^2} dx\right]$ 5

$= \dfrac{\pi}{2}(-1) + \pi \ln(1+x^2)\Big|_0^1$ 5

$= -\dfrac{\pi}{2} + \pi \ln(2)$

$= \dfrac{\pi}{2}\left[2\ln 2 - 1\right]$ // 5

$\boxed{25}$

(7) $x^3 - y^2 = 0$

$3x^2 - 2y . \dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{3x^2}{2y}$ 5

ස්පර්ශකයේ සමීකරණය

$y - 8t^3 = 3t(x - 4t^2)$

$3tx - y - 4t^3 = 0$ // 5

P හිදී අනුක්‍රමණය (ස්පර්ශකයේ) $= \dfrac{3}{2} \times \dfrac{16t^4}{8t^3}$

$= 3t$ 5

අභිලම්බයේ අනුක්‍රමණය $= \dfrac{-1}{3t}$

Q හිදී ස්පර්ශකයේ අනුක්‍රමණය $= 3T$

Q හිදී අභිලම්බයේ අනුක්‍රමණය $= -\dfrac{1}{3T}$ 5

$3t = -\dfrac{1}{3T}$

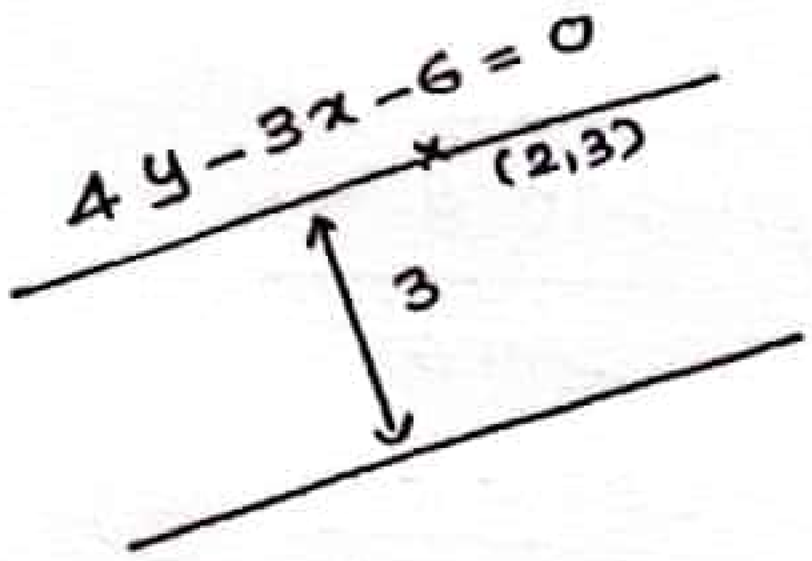
$T = -\dfrac{1}{9t}$ // 5

$\boxed{25}$

(8) $y - 3 = \frac{3}{4}(x-2)$

$4y - 12 = 3x - 6$ 5

$4y - 3x - 6 = 0$



l ට සමාන්තර සරල රේඛාවේ සමීකරණය

$4y - 3x + \lambda = 0$ 5 ; $\lambda \in \mathbb{R}$

$3 = \frac{|4(3) - 3(2) + \lambda|}{\sqrt{16+9}}$ 5

$15 = \pm(6+\lambda)$

$+/ \ \lambda = 9$ $\quad -/ \ \lambda = 21$ 5

සරල රේඛාවල සමීකරණ

$4y - 3x + 9 = 0$ 5

$4y - 3x + 21 = 0$

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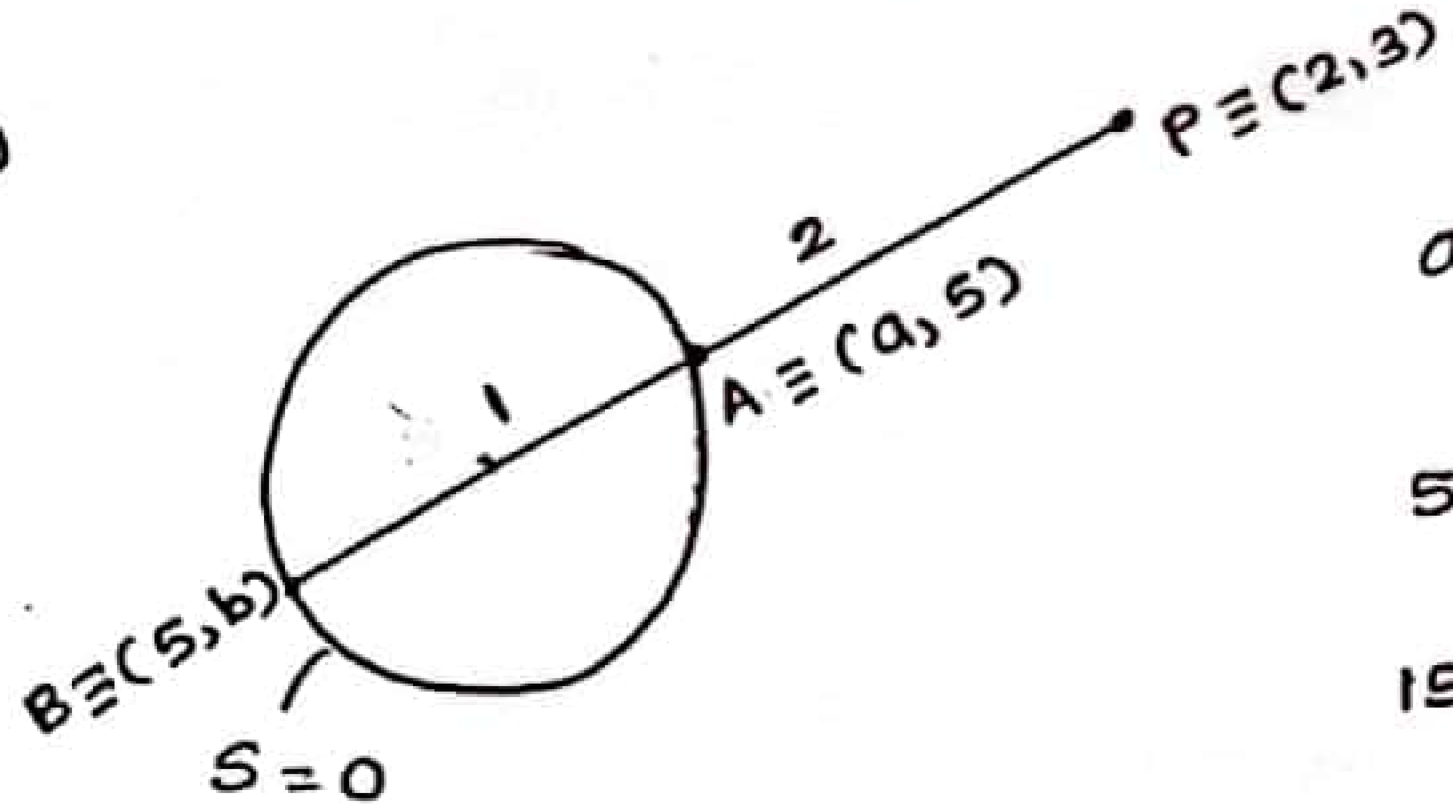
(9)



$a = \frac{10+2}{3} = 4$ 5

$5 = \frac{3+2b}{3}$

$15 = 3 + 2b$

$b = 6$ 5

$A \equiv (4,5)$ $\quad B \equiv (5,6)$

A හා B විෂ්කම්භයක අන්ත වූ වෘත්තයේ සමීකරණය 5

$(x-4)(x-5) + (y-5)(y-6) = 0$ 10

$x^2 - 9x + 20 + y^2 - 11y + 30 = 0$

$x^2 + y^2 - 9x - 11y + 50 = 0$

(10)

$$R.H.S = \frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}$$

$$= \frac{\sec\theta - 1}{\tan\theta} \quad 5$$

$$= \frac{\sec\theta}{\tan\theta} - \frac{1}{\tan\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1-\cos\theta}{\sin\theta} \quad 5$$

$$= \frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}$$

$$= \tan\theta/2 \quad 5$$

$$\tan\theta/2 = \frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}$$

5

$\theta = \pi/6$ ආදේශයෙන්

$$\tan\pi/12 = \frac{\sqrt{1+\tan^2\pi/6} - 1}{\tan\pi/6}$$

$$= \frac{\sqrt{1+(1/\sqrt{3})^2} - 1}{1/\sqrt{3}}$$

5

$$= 2 - \sqrt{3}$$

$\boxed{25}$

(11) $f(x) = x^2 - bx + c = 0$

$f(0) = c \neq 0$ 5 $(\because c \neq 0)$

$\therefore$ 0 මූලයක් නොවේ. 5

$\boxed{10}$

$\Delta = b^2 - 4(1)(c)$ 5

$= b^2 - 4c > 0$ 5 $(\because b^2 > 4c)$

$\therefore$ මූල තාත්වික හා ප්‍රභින්න වේ. 5

$\boxed{15}$

$\alpha + \beta = b$ 5 $\alpha\beta = c$

$(|\alpha| + |\beta|)^2 = \alpha^2 + \beta^2 + 2|\alpha|.|\beta|$ 5

$= (\alpha+\beta)^2 - 2\alpha\beta + 2|\alpha\beta|$

$= b^2 - 2c + 2|c|$ 5

$|\alpha| + |\beta| = \sqrt{b^2 - 2c + 2|c|}$ $(\because |\alpha| > 0$ හා $|\beta| > 0)$

$1 + \frac{1}{|\alpha|} + 1 + \frac{1}{|\beta|} = 2 + \frac{|\alpha| + |\beta|}{|\alpha\beta|}$ 5

$= 2 + \frac{\sqrt{b^2 - 2c + 2|c|}}{|c|}$

$= \frac{2|c| + \sqrt{b^2 - 2c + 2|c|}}{|c|}$ 5

$\left(1 + \frac{1}{|\alpha|}\right)\left(1 + \frac{1}{|\beta|}\right) = 1 + \frac{1}{|\alpha|} + \frac{1}{|\beta|} + \frac{1}{|\alpha|}\cdot\frac{1}{|\beta|}$ 5

$= 1 + \frac{\sqrt{b^2 - 2c + 2|c|}}{|c|} + \frac{1}{|c|}$

$$\left(1+\frac{1}{|\alpha|}\right)\left(1+\frac{1}{|\beta|}\right) = \frac{|c| + \sqrt{b^2-2c+2|c|} + 1}{|c|}$$ 5

$$x^2 - x\left(1+\frac{1}{|\alpha|} + 1 + \frac{1}{|\beta|}\right) + \left(1+\frac{1}{|\alpha|}\right)\left(1+\frac{1}{|\beta|}\right) = 0$$ 5

$$x^2 - x\left[\frac{2|c| + \sqrt{b^2-2c+2|c|}}{|c|}\right] + \left[\frac{|c|+1+\sqrt{b^2-2c+2|c|}}{|c|}\right] = 0$$ 5

$$|c|x^2 - x\left(\sqrt{b^2-2c+2|c|} + 2|c|\right) + \sqrt{b^2-2c+2|c|} + |c| + 1 = 0$$ 5

$\alpha, \beta > 0$ හෝ $\alpha, \beta < 0 \Rightarrow \alpha\beta > 0$ වේ. 5

$$\Rightarrow c > 0$$

$$\therefore |c| = c$$ 5

$\left(1+\frac{1}{|\alpha|}\right)$ හා $\left(1+\frac{1}{|\beta|}\right)$ මූලවන සමීකරණය

$$cx^2 - x\left(\sqrt{b^2-2c+2c} + 2c\right) + \sqrt{b^2-2c+2c} + c + 1 = 0$$ 5

$$cx^2 - x\left(|b| + 2c\right) + \left(|b| + c + 1\right) = 0$$

$\boxed{65}$

(b) $2x^3 + ax^2 + bx + c \equiv g(x)(x^2-1) + 6x - 3$ 5

$x = 1$ ආදේශයෙන්

$2 + a + b + c = 3 \longrightarrow ①$ 5

$x = -1$ ආදේශයෙන්

$-2 + a - b + c = -9 \longrightarrow ②$ 5

① − ② $\quad 2b + 4 = 12$

$2b = 8$

$b = 4$ 5

$2x^3 + ax^2 + bx + c = \phi(x) \cdot x(x-3) + kx + 4$ 5

$x = 0$ ආදේශයෙන්

$c = 4$ 5

$x = 3$

$54 + 9a + 3b + c = 3k + 4$

$9a + 3b + c = 3k - 50$

$9a + 12 = 3k - 54$

$9a = 3k - 66 \longrightarrow ③$ 5

①න් $\quad a = -7$ 5

③න් $\quad 3k = 3$

$k = 1$ 5

$h(x) = 2x^3 - 7x^2 + 4x + 4$

$h(2) = 16 - 28 + 8 + 4$

$= 28 - 28$

$= 0$ 5

$\therefore (x-2)$, $h(x)$ හි සාධකයකි.

$h(x) = 2x^3 - 7x^2 + 4x + 4$

$= (x-2)(2x^2 - 3x - 2)$

$= (x-2)(x-2)(2x+1)$ 10

$= (x-2)^2(2x+1)$

$p = 2 \quad q = -1$

$\boxed{60}$

(12)

පිරිමි (8)	ගැහැණු (12)
5	5

5

$^8C_5 \times {}^{12}C_5 = \dfrac{6\times7\times8}{2\times3} \times \dfrac{8\times9\times10\times11\times12}{2\times3\times4\times5}$

$= 712 \times 56$ 5

$= 44352$//

(ii)

පිරිමි	ගැහැණු
5	5
6	4
7	3

$^8C_5 \times {}^{12}C_5 = 44352$

$^8C_6 \times {}^{12}C_4 = 13860$ 10

$^8C_7 \times {}^{12}C_3 = 1760$ 10

$\underline{\underline{59,972}}$ 5

(iii)

D – M (2)	D – F (4)	E – M (4)	E – F (4)	A – M (2)	A – F (4)	ආකාර
2	1	2	1	2	2	$^2C_2\times{}^4C_1\times{}^4C_2\times{}^4C_1\times{}^2C_2\times{}^4C_2 = 576$ 10
2	1	2	2	2	1	$^2C_2\times{}^4C_1\times{}^4C_2\times{}^4C_2\times{}^2C_2\times{}^4C_1 = 576$ 10
2	2	2	1	2	1	$^2C_2\times{}^4C_2\times{}^4C_2\times{}^4C_1\times{}^2C_2\times{}^4C_1 = 576$ 10

මුළු ආකාර $= 576 \times 3$

$= 1728$// 5

$\boxed{70}$

$$u_r = f(r) - f(r+1)$$

(b) $$\frac{1}{r(r+1)(r+2)} = \frac{\lambda}{r(r+1)} - \frac{\lambda}{(r+1)(r+2)} \quad 5$$

$$1 = \lambda(r+2) - \lambda r \quad 5$$

[illegible]/ $$1 = 2\lambda$$

$$\lambda = \tfrac{1}{2} \quad 5$$

$$f(r) = \frac{1}{2r(r+1)} \quad 5$$

$$u_r = f(r) - f(r+1)$$

$r=1$ $\quad u_1 = f(1) - \cancel{f(2)}$

$r=2$ $\quad u_2 = \cancel{f(2)} - \cancel{f(3)} \quad 5$

$\vdots$

$r=n-1$ $\quad u_{n-1} = \cancel{f(n-1)} - \cancel{f(n)} \quad 5$

$r=n$ $\quad u_n = \cancel{f(n)} - f(n+1)$

$$\sum_{r=1}^{n} u_r = f(1) - f(n+1) \quad 5$$

$$= \frac{1}{2(2)} - \frac{1}{2(n+1)(n+2)}$$

$$\sum_{r=1}^{n} u_r = \frac{1}{4} - \frac{1}{2(n+1)(n+2)} \quad 5 \qquad \boxed{40}$$

$$V_r = u_{r+1}$$

$$\sum_{r=1}^{n} V_r = \sum_{r=1}^{n} u_{r+1}$$

$$= \sum_{r=1}^{n+1} u_r - u_1 \quad 5$$

$$\sum_{r=1}^{n} V_r = \frac{1}{4} - \frac{1}{2(n+2)(n+3)} - \frac{1}{6} \quad 5$$

$$= \frac{1}{12} - \left[\frac{1}{2(n+2)(n+3)}\right]$$

$$= \frac{1}{12} - \frac{1}{2(n+2)(n+3)} \quad 5$$

$$U_r + V_r = \frac{1}{r(r+1)(r+2)} + \frac{1}{(r+2)(r+1)(r+3)}$$

$$W_r = \frac{2r+3}{r(r+1)(r+2)(r+3)}$$

$$\sum_{r=1}^{n} W_r = \sum_{r=1}^{n}(U_r + V_r) = \sum_{r=1}^{n} U_r + \sum_{r=1}^{n} V_r$$

$$= \frac{1}{4} - \frac{1}{2(n+1)(n+2)} + \frac{1}{12} - \frac{1}{2(n+2)(n+3)} \quad 5$$

$$= \frac{1}{3} - \frac{1}{2(n+1)(n+2)} - \frac{1}{2(n+2)(n+3)} \quad 5$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} W_r = \lim_{n\to\infty} \frac{1}{3} - \lim_{n\to\infty}\left(\frac{1}{2(n+1)(n+2)} + \frac{1}{2(n+2)(n+3)}\right) \quad 5$$

$$= \frac{1}{3} \quad 5$$

ශ්‍රේණිය අභිසාරී වේ. ලෝකාන්තය $1/3$ වේ. 5

$\boxed{40}$

$\boxed{70} + \boxed{80} = \boxed{150}$

(13) (a) $A^T B = C$

$$\begin{pmatrix} a & 2 & b \\ 0 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 4 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$$ (5)

$$\begin{pmatrix} a+8+2b & 5a-2+b \\ -8+6 & 2+3 \end{pmatrix} = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$$

$$\begin{pmatrix} a+2b+8 & 5a+b-2 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$$ (5)

$a+2b+8 = 15 \rightarrow ①$ $\quad 5a+b-2 = 6 \rightarrow ②$ $\quad -2 = c \rightarrow ③$

සමීකරණ ③ ටම → (10)

$c = -2$ // (5)

① න් $\quad a+2b = 7 \rightarrow ④$

② න් $\quad 5a+b = 8 \rightarrow ⑤$

④ − 5×2 $\quad -9a = -9$

$a = 1$ // (5) $\quad b = 3$ // (5) $\quad \boxed{35}$

$$C = \begin{pmatrix} 15 & 6 \\ -2 & 5 \end{pmatrix}$$

$\det(C) = 15\times5 - (-2)\times6$

$= 75+12$

$= 87$ (5)

$\det(C) \neq 0$ නිසා C^{-1} පවතී. (5) $\quad \boxed{10}$

$$C^{-1} = \frac{1}{87}\begin{pmatrix} 5 & -6 \\ 2 & 15 \end{pmatrix} \quad 5 \quad (10)$$

$$C(P+2I) = 3C + I.$$

$$C^{-1}C(P+2I) = 3C^{-1}C + C^{-1}I \quad 5$$

$$P+2I = 3I + C^{-1}I$$

$$P = I + C^{-1} \quad 5$$

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{87}\begin{pmatrix} 5 & -6 \\ 2 & 15 \end{pmatrix}$$

$$P = \frac{1}{87}\begin{pmatrix} 92 & -6 \\ 2 & 102 \end{pmatrix} \quad 5$$

$\boxed{65}$

(b) $z\bar{z} = |z|^2$

$z = x + iy$ ලෙස ගනිමු. $x, y \in \mathbb{R}$ වේ.

L.H.S $= z\bar{z}$

$= (x+iy)(x-iy)$ 5

$= x^2 + y^2$

$= |z|^2$ 5

$\boxed{10}$

$$|z-2i|^2 = (z-2i)(\overline{z-2i}) \quad 5$$

$$= (z-2i)(\bar{z}+2i) \quad 5$$

$$= z\bar{z} + 2i(z-\bar{z}) - 4i^2 \quad 5$$

$$= |z|^2 + 2i\,\mathrm{Im}(z).2i + 4$$

$$= |z|^2 - 4\,\mathrm{Im}(z) + 4 \,// \quad 5$$

$$|1+2iz|^2 = (1+2iz)(\overline{1+2iz})$$

$$= (1+2iz)(1-2i\bar{z}) \quad 5$$

$$= 1 + 2i(z-\bar{z}) - 4i^2 z\bar{z}$$

$$= 1 + 2i.\,2i\,\mathrm{Im}(z) + 4|z|^2$$

$$= 1 - 4\,\mathrm{Im}(z) + 4|z|^2 \,// \quad 5$$

$\boxed{30}$

(ii) $$\left|\frac{1+2iz}{z-2i}\right| = 1$$

$$\Leftrightarrow |1+2iz| = |z-2i| \quad 5$$

$$\Leftrightarrow |1+2iz|^2 = |z-2i|^2$$

$$\Leftrightarrow 1 - 4\,\mathrm{Im}(z) + 4|z|^2 = |z|^2 - 4\,\mathrm{Im}(z) + 4$$

$$\Leftrightarrow 3|z|^2 = 3$$

$$\Leftrightarrow |z|^2 = 1$$

$$|z| = 1 \,// \quad 5$$

$\boxed{10}$

$$\left|\frac{1+2iz}{z-2i}\right| = 1 \quad \text{නම්} \qquad |z| = 1$$

$$\text{Arg}(2iz) = \frac{\pi}{6}$$

$$\text{Arg}(2i) + \text{Arg}(z) = \pi/6 \quad 5$$

$$\frac{\pi}{2} + \text{Arg}(z) = \pi/6$$

$$\text{Arg}(z) = \frac{\pi}{6} - \frac{\pi}{2}$$

$$\text{Arg}(z) = -\frac{2\pi}{3} \quad 5$$

$$z = (\cos(-2\pi/3) + i\sin(-2\pi/3)) \quad 5$$

$$z = \left(\cos\frac{2\pi}{3} - i\sin(2\pi/3)\right) \qquad \boxed{15}$$

(c) $z = \sqrt{6} + \sqrt{2}i = \sqrt{2}(\sqrt{3} + i)$

$$z = 2\sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) \quad 5$$

$$= 2\sqrt{2}\left[\cos\pi/6 + i\sin\pi/6\right] \quad 5$$

$$r = 2\sqrt{2} \qquad \theta = \pi/6$$

$$(\sqrt{6} + \sqrt{2}i)^6 = (2\sqrt{2})^6\,[\cos\pi + i\sin\pi] \quad 5$$

$$= 512(-1)$$

$$= -512 \quad 5 \qquad \boxed{20}$$

(14) $f(x) = \dfrac{2x-1}{(x-1)^2}$

$$f'(x) = \frac{(x-1)^2 . 2 - (2x-1) \cdot 2(x-1)}{(x-1)^4}$$ 15

$$= \frac{(x-1)\left(2(x-1) - 2(2x-1)\right)}{(x-1)^4}$$

$$f'(x) = \frac{2(-x)}{(x-1)^3}$$ 5

$\boxed{25}$

$A = -2$ // 5

හැරුම් ලක්ෂ සඳහා $f'(x) = 0$

$f'(x) = 0 \Leftrightarrow x = 0$ 5

$\lim\limits_{x \to \pm\infty} f(x) = 0$

$f(x) = 0$ තිරස් ස්පර්ශෝන්මුඛයකි. 5

$\lim\limits_{x \to 1^-} f(x) = \infty$

$\lim\limits_{x \to 1^+} f(x) = \infty$

$x = 1$ සිරස් ස්පර්ශෝන්මුඛයකි. 5

	$-\infty < x < 0$	$0 < x < 1$	$1 < x < \infty$
$f'(x)$ ලකුණ	$-$ අඩුවේ 5	$+$ වැඩිවේ 5	$-$ අඩුවේ. 5

$x = 0$ $x \neq 1$

අඩුවන ප්‍රාන්තර : $(-\infty, 0]$ හා $(1, \infty)$ } 5

වැඩිවන ප්‍රාන්තර : $[0, 1)$

$(0, -1)$ දී ස්ථානීය අවමයක් ඇත. 5

$f''(x) = 0 \iff x = -\frac{1}{2}$ 5

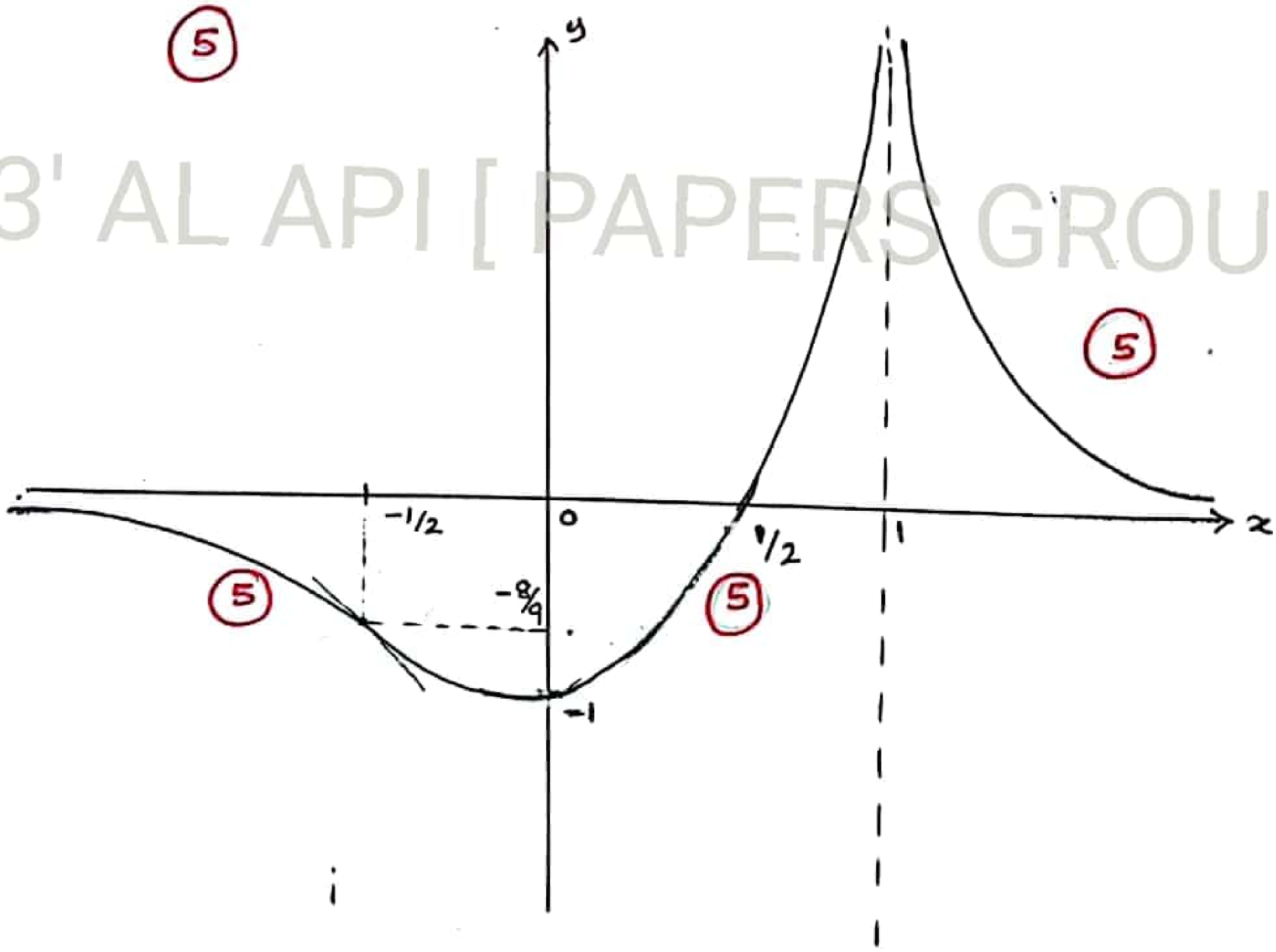
	$-\infty < x < -\frac{1}{2}$	$-\frac{1}{2} < x < 1$	$1 < x < \infty$
$f''(x)$ ලකුණ	$-$ යටි අවතල	$+$ උඩු අවතල	$+$ උඩු අවතල

$x = -\frac{1}{2}$ $\qquad$ $x \neq 1$

10

$(-\frac{1}{2}, -\frac{8}{9})$ දී නතිවර්තන ලක්ෂයක් ඇත.

5



75

(b) $2P = (x+y)2$ 5

$P = x+y$

$y = P - x$ 5

$A = xy - 4\times\frac{1}{2}\times\frac{x^2}{4}\times\frac{\pi}{2}$ 5

$= x(P-x) - \frac{\pi x^2}{4}$

$A = Px - \left(\frac{\pi+4}{4}\right)x^2$ 5

$\frac{dA}{dx} = P - \left(\frac{\pi+4}{4}\right)\cdot 2x$ 5

$\frac{dA}{dx} = 0 \Leftrightarrow x = \frac{2P}{(\pi+4)}$ 5

	$0 < x < \frac{2P}{\pi+4}$	$x > 2P/\pi+4$
$\frac{dA}{dx}$ ලකුණ	$-$	$+$

5

$\therefore x = \frac{2P}{\pi+4}$ විට A උපරිම වේ. 5

එවිට $y = P - \frac{2P}{\pi+4}$

$= \frac{P(2+\pi)}{\pi+4}$ 5

$x : y = 2 : (\pi+2)$ 5

$\boxed{50}$

(15) $\dfrac{x}{(x+1)(x^2+4)} = \dfrac{A}{(x+1)} + \dfrac{Bx+C}{x^2+4}$ (5)

$x = A(x^2+4) + (Bx+C)(x+1)$

$x^2/$ $\quad 0 = A+B$

$x/$ $\quad 1 = B+C$

$x^0/$ $\quad 0 = 4A+C$

$A = -1/5$ (5) $\quad B = 1/5$ (5) $\quad C = 4/5$ (5) $\quad \boxed{20}$

$\dfrac{x}{(x+1)(x^2+4)} = \dfrac{-1}{5(x+1)} + \dfrac{1(x+4)}{5(x^2+4)}$ (5)

$\displaystyle\int \frac{x}{(x+1)(x^2+4)}\,dx = -\frac{1}{5}\int\frac{1}{(x+1)}\,dx + \frac{1}{5}\int\frac{x+4}{x^2+4}\,dx$ (5)

$\displaystyle = -\frac{1}{5}\ln|x+1| + \frac{1}{10}\int\frac{2x}{x^2+4}\,dx + \frac{4}{5}\int\frac{1}{x^2+(2)^2}\,dx$ (5)

$\displaystyle = -\frac{1}{5}\ln|x+1| + \frac{1}{10}\ln(x^2+4) + \frac{4}{5}\times\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

$\displaystyle = -\frac{1}{5}\ln|x+1| + \frac{1}{10}\ln(x^2+1) + \frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right) + c$ (5) (5) (5) (5)

C යනු අභිමත නියතයකි. ලේඛයෙම

$\boxed{35}$

(b) $\displaystyle\int \frac{x \sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$\frac{dv}{dx} = x(1-x^2)^{-1/2}$

$\frac{dv}{dx} = (-2x(1-x^2)^{-1/2})\left(-\frac{1}{2}\right)$

$v = -(1-x^2)^{1/2}$

5

$u = \sin^{-1}x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$I = \sin^{-1}x\,.\,(-\sqrt{1-x^2}) + \int (1-x^2)^{1/2}\,.\,\frac{1}{\sqrt{1-x^2}}\,dx$

5

$I = -\sin^{-1}x\sqrt{1-x^2} + \int 1\,dx$

$I = -\sin^{-1}x\,.\,\sqrt{1-x^2} + x + C$ 5 පේළියටම

5

C යෙදූ අභිමත නියතයකි.

$\boxed{20}$

(c) $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ බව.

R.H.S

$= \displaystyle\int_0^a f(a-x)\,dx$

$a-x = t$ ලෙස ගනිමු.

$dx = -dt$

$x = 0$ දී $\quad t = a$

$x = a$ දී $\quad t = 0$

5

$= \displaystyle\int_a^0 f(t)\,.\,(-dt)$

$= \displaystyle\int_0^a f(t)\,.\,dt$

නිශ්චිත අනුකල විචල්‍යයෙන් ස්වායත්ත නිසා.

5

$\boxed{10}$

$= \displaystyle\int_0^a f(x)\,.\,dx \quad = R.H.S$

$$I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx$$

$$= \int_0^{\pi/2} \frac{(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}\, dx \quad (5)$$

$$= \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x}\, dx$$

$$= \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x}\, dx - \int \frac{x}{\cos x + \sin x}\, dx \quad (5)$$

$$I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x}\, dx - I$$

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x}\, dx$$

$\boxed{15}$

$$I = \frac{\pi}{4}\int_0^{\pi/2} \frac{1}{\cos x + \sin x}\, dx \quad (5)$$

$$I = \frac{\pi}{4}\int_0^{\pi/2} \frac{1}{\cos x + \sin x}\, dx$$

$t = \tan x/2$ ආදේශයෙන.

$dt = \sec^2 x/2 \cdot 1/2 \cdot dx.$

$$dx = \frac{2\,dt}{1 + \tan^2 x/2}$$

$$dx = \frac{2\,dt}{1+t^2}. \quad (5)$$

$$\sin x = \frac{2t}{1+t^2} \quad (5)$$

$$\cos x = \frac{1-t^2}{1+t^2}. \quad (5)$$

$t = \tan x/2.$

$x = 0$ දී $t = 0$

$x = \frac{\pi}{2}$ දී $t = 1$ (5)

$$I = \frac{\pi}{4}\int_0^1 \frac{1}{\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2}} \cdot \frac{2\,dt}{1+t^2}. \quad (5)$$

$$I = \frac{\pi}{4}\int_0^1 \frac{2\,dt}{-t^2 + 2t + 1}$$

$$I = \frac{\pi}{2}\int_0^1 \frac{dt}{(\sqrt{2})^2 - (t-1)^2}\, dt$$

$$\frac{1}{(\sqrt{2}-t+1)(\sqrt{2}+t-1)} = \frac{A}{(\sqrt{2}-t+1)} + \frac{B}{(\sqrt{2}+t-1)}$$

$$1 = A(\sqrt{2}+t-1) + B(\sqrt{2}-t+1)$$

සංගුණක සැසඳීම.

$t /\quad 0 = A - B \rightarrow ① \qquad t^0 /\quad 1 = (\sqrt{2}-1)A + (\sqrt{2}+1)B \rightarrow ②$

① න් $\quad A = B$

② න් $\quad (\sqrt{2}-1)A + (\sqrt{2}+1)A = 1$

$$2\sqrt{2}A = 1 \qquad 10$$

$$A = \frac{1}{2\sqrt{2}} \qquad B = \frac{1}{2\sqrt{2}}.$$

$$I = \frac{\pi}{2}\left[\int_0^1 \frac{1}{2\sqrt{2}(\sqrt{2}+1-t)}\,dt + \int_0^1 \frac{1}{2\sqrt{2}(\sqrt{2}+t-1)}\,dt\right]$$

$$= \frac{\pi}{4\sqrt{2}}\left[-\int_0^1 \frac{-1}{(\sqrt{2}+1-t)}\,dt + \int_0^1 \frac{1}{\sqrt{2}+t-1}\,dt\right]$$

$$= \frac{\pi}{4\sqrt{2}}\left[-\ln|\sqrt{2}+1-t|\Big|_0^1 + \ln|\sqrt{2}+t-1|\Big|_0^1\right]$$

$$5 \qquad 5$$

$$= \frac{\pi}{4\sqrt{2}}\left[-\ln(\sqrt{2}) + \ln(\sqrt{2}) + \ln(\sqrt{2}+1) - \ln(\sqrt{2}-1)\right]$$

$$= \frac{\pi}{4\sqrt{2}}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$$

$$= \frac{\pi}{4\sqrt{2}}\ln(\sqrt{2}+1)^2$$

$$5$$

$$= \frac{\pi}{2\sqrt{2}}\ln(\sqrt{2}+1)$$

50

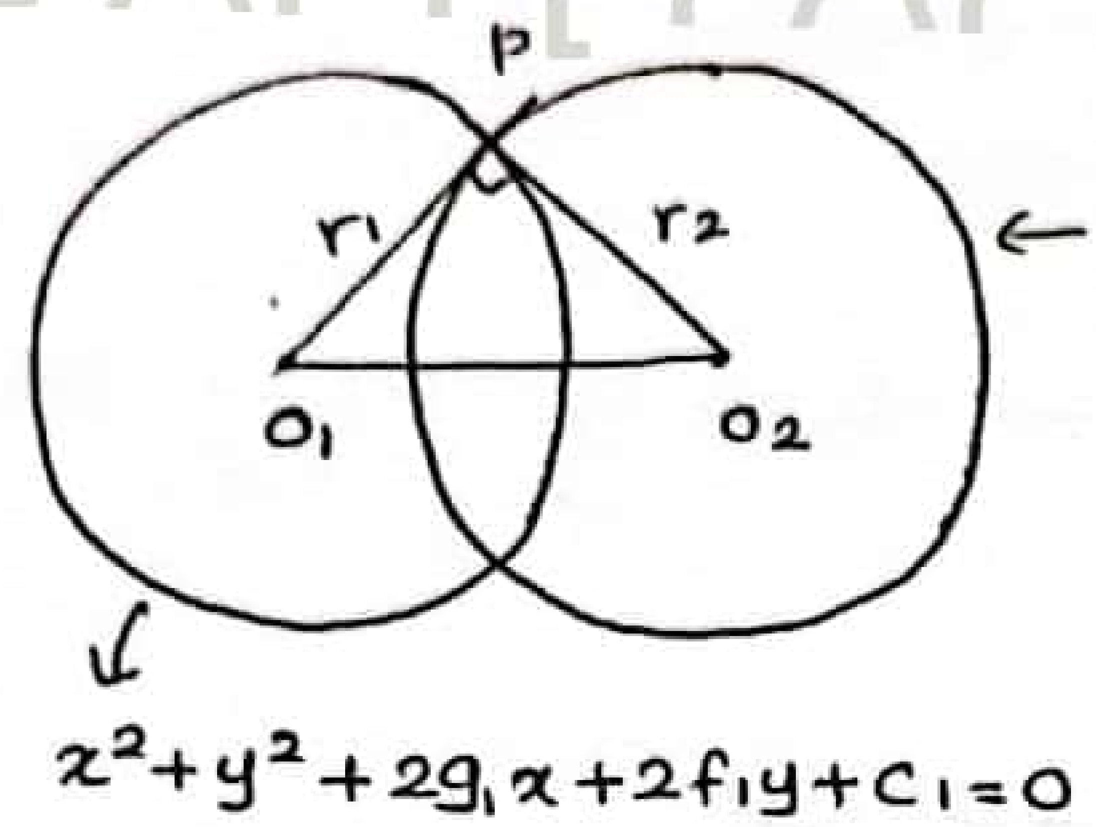
(16) $(3a - 8a + 15a)(6a - 16a + 15a) > 0$

$(10a)(5a) > 0 \quad (a \neq 0)$

∴ A හා B ලක්ෂ රේඛාවෙහි (5) එකම පැත්තේ පිහිටයි. (5)

$\boxed{15}$

(b)



$O_1 \equiv (-g_1, -f_1)$

$O_2 \equiv (-g_2, -f_2)$

වෘත්ත දෙක ප්‍රලම්බව ඡේදනය වන්නේ නම්,

$(O_1O_2)^2 = (PO_1)^2 + (PO_2)^2$ (5)

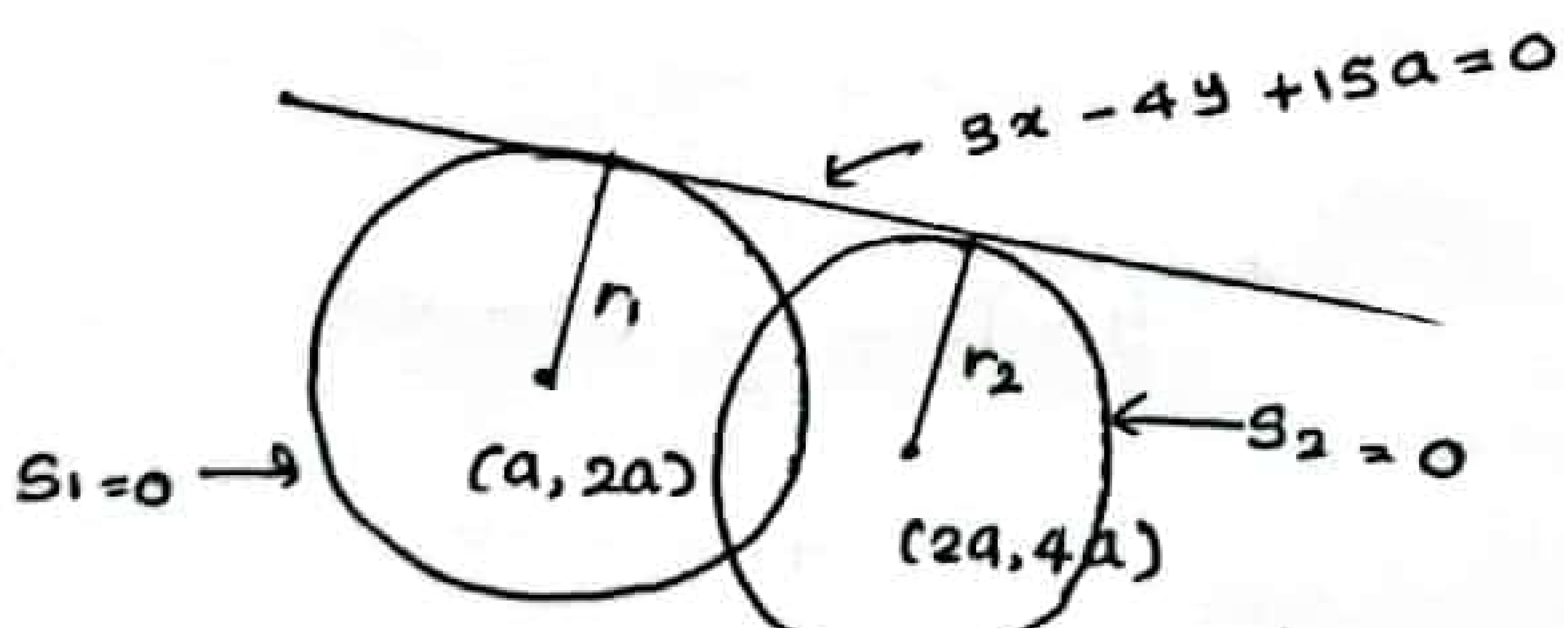
$(-g_1+g_2)^2 + (-f_1+f_2)^2 = r_1^2 + r_2^2$ (10)

$(-g_1+g_2)^2 + (-f_1+f_2)^2 = g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2$

$-2g_1g_2 - 2f_1f_2 = -c_1 - c_2$

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$ (5)

$\boxed{20}$



$\sqrt{a^2+4a^2-c_1} = \frac{|3a-8a+15a|}{5}$ (5) (5)

$25(5a^2-c_1) = 100a^2$

$5a^2 - c_1 = 4a^2$

$a^2 = c_1$ (5)

$S_1 = x^2+y^2-2ax-4ay+a^2=0$ (5)

$\sqrt{4a^2+16a^2-c_2} = \frac{|6a-16a+15a|}{5}$ (5) (5)

$25(20a^2-c_2) = 25a^2$

$20a^2 - c_2 = a^2$

$19a^2 = c_2$ (5)

$S_2 = x^2+y^2-4ax-8ay+19a^2=0$ (5)

$\boxed{40}$

$$2g_1g_2 + 2f_1f_2 = 2(-a)(-2a) + 2(-2a)(-4a) \quad 5 \quad 5$$

$$= 4a^2 + 16a^2$$

$$= 20a^2 \quad 5$$

$$c_1 + c_2 = 19a^2 + a^2$$

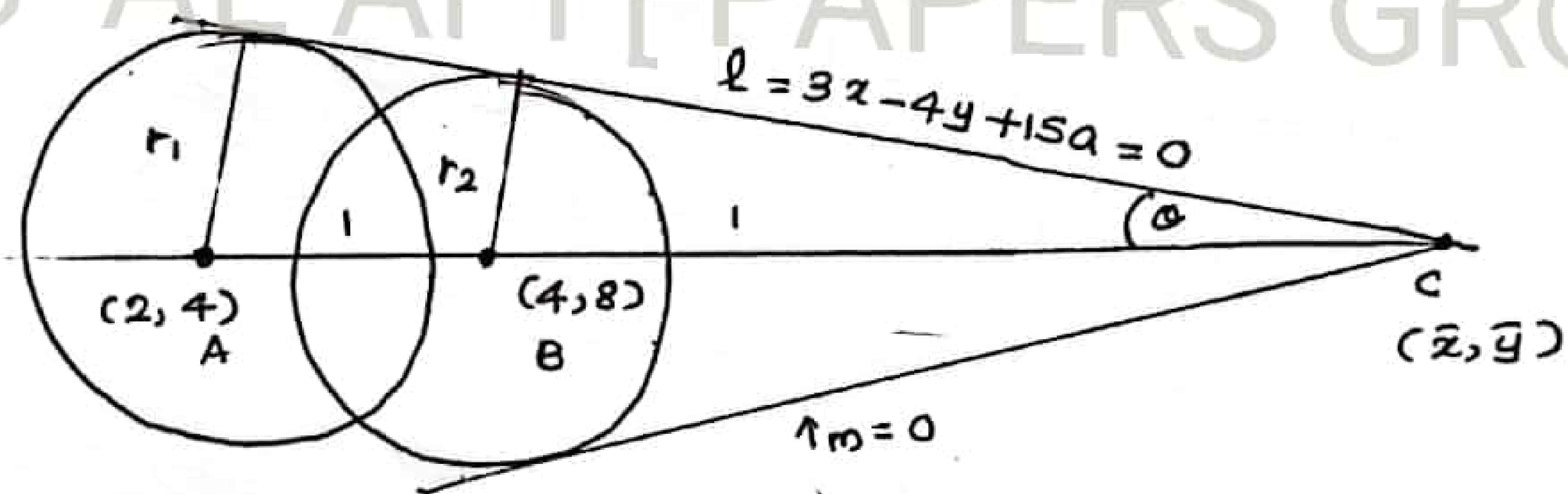
$$= 20a^2 \quad 5$$

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$ නිසා $S_1 = 0$ හා $S_2 = 0$ වෘත්ත 5

ප්‍රලම්බව ඡේදනය වේ.

$$S_1 = x^2 + y^2 - 4x - 8y + 4 = 0 \quad 5$$

$$S_2 = x^2 + y^2 - 8x - 16y + 76 = 0 \quad 5$$



$$r_1 = \sqrt{4+16-4} = 4$$

$$r_2 = \sqrt{16+64-76} = 2$$

$$r_1 : r_2 = 2:1 \quad 5$$

$$4 = \frac{\bar{x}+2}{2} \qquad 8 = \frac{\bar{y}+4}{2}$$

$$\bar{x} = 6 \qquad \bar{y} = 12$$

$$C \equiv (6, 12) \quad 10$$

ස්පර්ශකයේ අනුක්‍රමණය $= m$

ස්පර්ශකයේ සමීකරණය

$y - 12 = m(x - 6)$ 5

$mx - y + 12 - 6m = 0$

5

$$\frac{|m(2) - 4 + 12 - 6m|}{\sqrt{m^2+1}} = 4$$

$= |8 - 4m| = 4\sqrt{m^2+1}$

$= (2 - m)^2 = m^2 + 1$

$= 4m = 3$

$= m = 3/4$ 5

මෙය දෙන ලද ස්පර්ශකයේ අනුක්‍රමණයයි.

අනෙක් ස්පර්ශකය y අක්ෂයට සමාන්තර වේ. 5

$\therefore$ $x = 6$ ස්පර්ශකයේ සමීකරණය වේ.

5

$\boxed{75}$

(17) $f(x) = \dfrac{1 + \cot x}{1 + \cot^2 x}$

$$= \frac{(\cos x + \sin x)\sin x}{\sin^2 x + \cos^2 x}$$ 5

$= (2\cos x \sin x + 2\sin^2 x)\frac{1}{2}$

$= \frac{1}{2}(\sin 2x + 1 - \cos 2x)$ 5

$= 1\left(\frac{1}{2}\sin 2x - \cos 2x \cdot \frac{1}{2}\right) + \frac{1}{2}$

$= -\frac{1}{2}\left(\cos 2x \cos\frac{\pi}{3} - \sin 2x \sin\frac{\pi}{3}\right) + \frac{1}{2}$ $\boxed{15}$

5

$= -\frac{1}{2}\cos(2x + \pi/3) + 1/2$

$A = -1/2$ $B = 1/2$

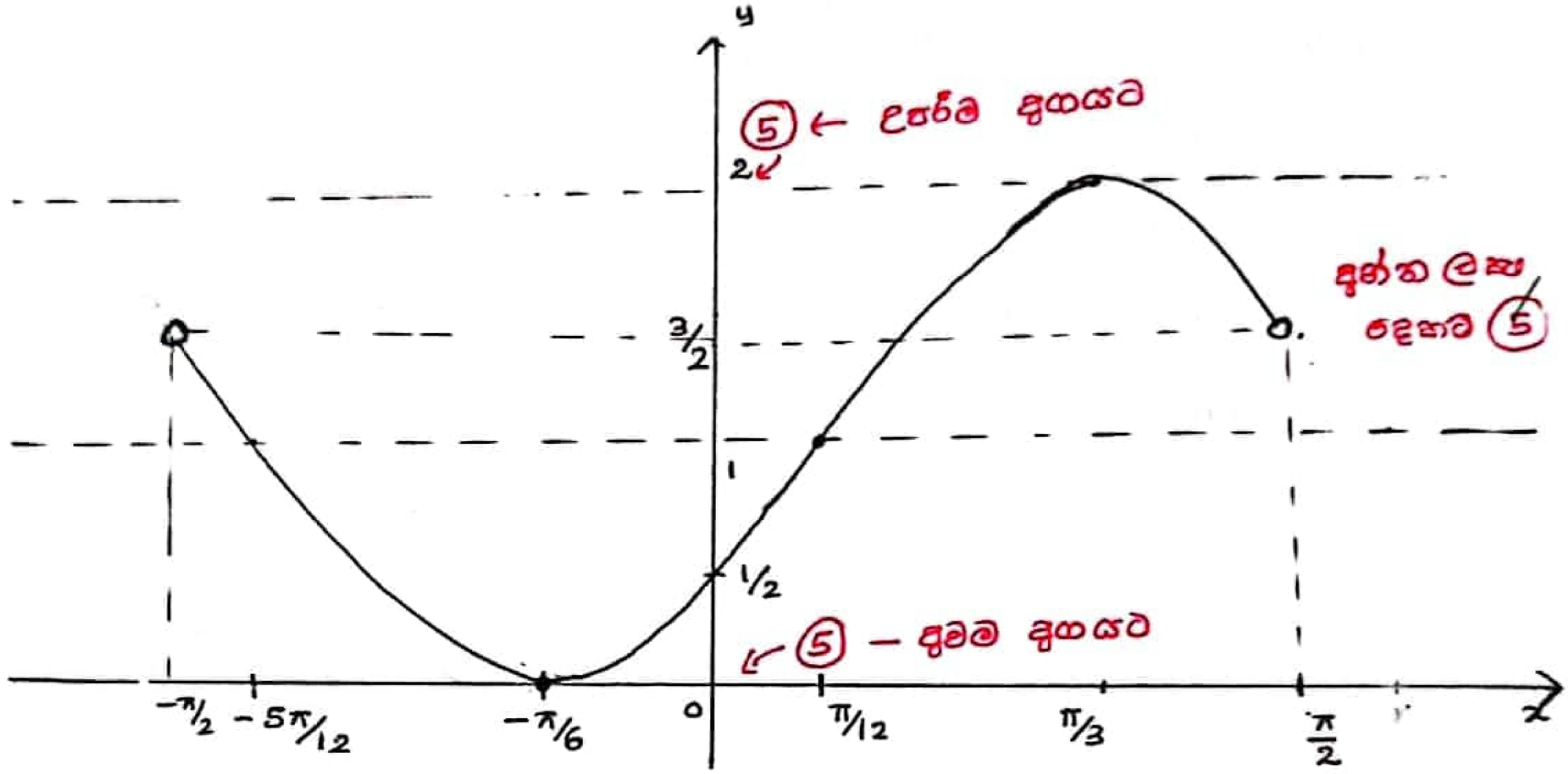
$\alpha = \pi/3$

$y = 2f(x)$

$= -\cos\left(2x + \pi/3\right) + 1$ (5)

ආවර්තය $= \pi$ x අක්ෂය ඔස්සේ විස්ථාපනය $\Rightarrow x = -\pi/6$ උපරිම අගය $= 2$

අවම අගය $= 0$



නිවැරදි ප්‍රස්තාරයට — (10)

(හැඩයට – නිවැරදි ඛණ්ඩාංක වලට)

$x = 0$ දී $y = -\cos \pi/3 + 1 = 1/2$

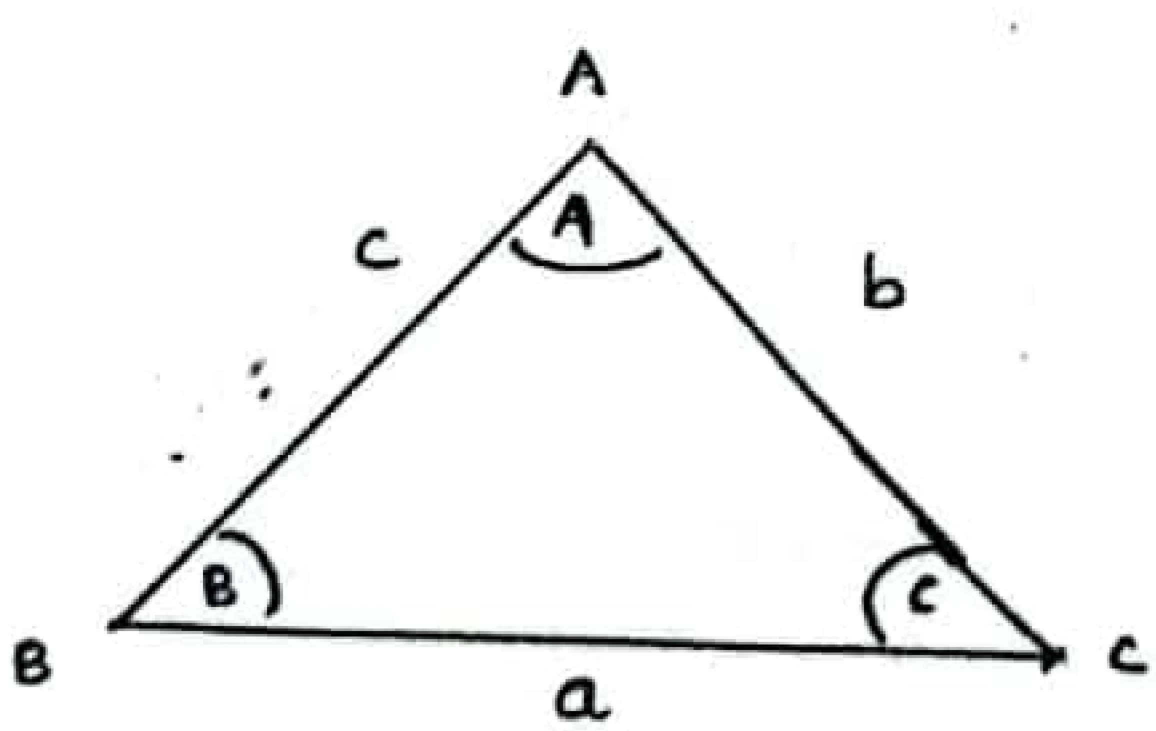
$x = \pi/2$ දී $y = -\cos\left(\pi + \pi/3\right) + 1 = 1/2 + 1 = 3/2$

$x = -\pi/2$ දී $y = -\cos\left(-\pi + \pi/3\right) + 1 = +1/2 + 1 = 3/2$

අගයන් තුනටම (5)

$\boxed{35}$

(b)



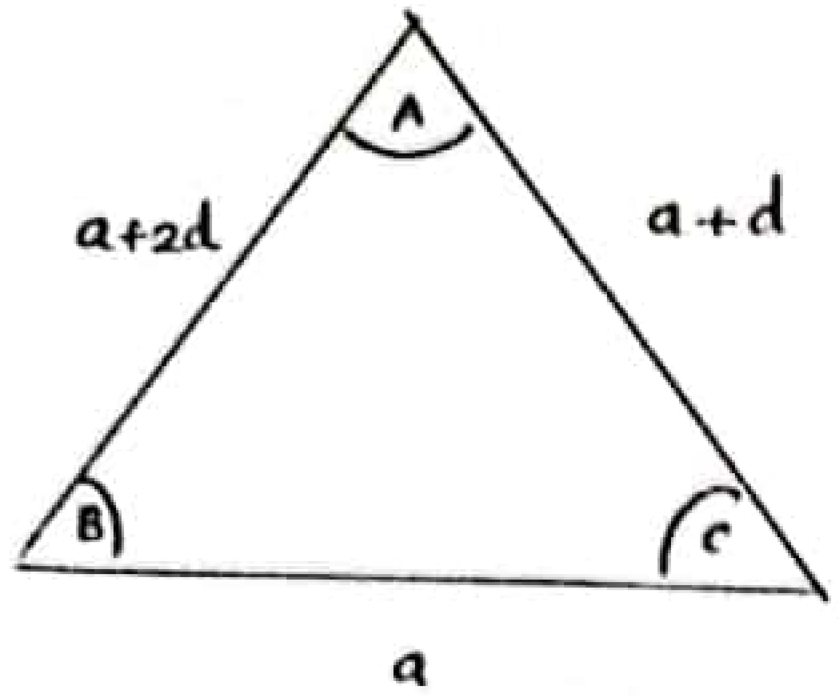
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$ (10)

සාධනයට — (20)

සුළු ∠ Δ මහා ∠ Δ සෘජු ∠ Δ

(10) + (5) + (5)

$\boxed{30}$



$$\cos C = \frac{a^2 + (a+d)^2 - (a+2d)^2}{2a(a+d)} \quad 5$$

$$= \frac{a^2 - 3d^2 - 2ad}{2a(a+d)}$$

$$= \frac{(a+d)^2 - 4d^2 - 4ad}{2a(a+d)}$$

$$= \frac{a+d}{2a} - \frac{4d(d+a)}{2a(a+d)} \quad 5$$

$$= \frac{a+d}{2a} - \frac{2d}{a}$$

$$= \frac{a+d-4d}{2a}$$

$$\cos C = \frac{1}{2} - \frac{3d}{2a} \quad 5$$

$$\frac{2\pi}{3} < C < \pi$$

$$\cos 2\pi/3 > \cos C > \cos \pi \quad 5$$

$$-1/2 > 1/2 - 3d/2a > -1$$

$$-1 > -\frac{3d}{2a} > -\frac{3}{2}$$

$$\frac{2}{3} < \frac{d}{a} < 1 \quad 5$$

$\boxed{25}$

(c) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = \pi^3 a$

$\alpha^3 + \beta^3 = \pi^3 a$ 5

$\sin^{-1}x = \alpha$ $\quad$ $\sin^{-1}x = \beta$

$\sin\alpha = x$ $\quad$ $\sin\beta = x$

$$\alpha + \beta = \frac{\pi}{2} \quad 5$$

$$(\alpha+\beta)^3 = \frac{\pi^3}{8}$$

$$\alpha^3 + \beta^3 + 3\alpha\beta(\alpha+\beta) = \frac{\pi^3}{8} \quad 5$$

$$\pi^3 a + 3\alpha\beta(\alpha+\beta) = \frac{\pi^3}{8}.$$

$$3\alpha\beta \cdot \left(\frac{\pi}{2}\right) = \frac{\pi^3}{8}\left[1 - 8a\right] \quad 5$$

$$\alpha\beta = \frac{\pi^2}{12}(1-8a)$$

$$\alpha\left(\frac{\pi}{2}-\alpha\right) = \frac{\pi^2}{12}(1-8a) \quad 5$$

$$\alpha^2 - \frac{\pi}{2}\alpha = (8a-1)\frac{\pi^2}{12}$$

$$\alpha^2 - \frac{\pi}{2}\alpha + \frac{\pi^2}{16} = (8a-1)\cdot\frac{\pi^2}{12} + \frac{\pi^2}{16} \quad 5$$

$$\left(\alpha - \frac{\pi}{4}\right)^2 = \frac{\pi^2}{48}\left(32a - 4 + 3\right)$$

$$\underline{\underline{\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 = \frac{\pi^2}{48}(32a-1)}} \quad 5$$

$$\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 \geq 0 \quad 5$$

$$\frac{\pi^2}{48}(32a-1) \geq 0$$

$$a \geq \frac{1}{32} \quad 5$$

$$\boxed{45}$$